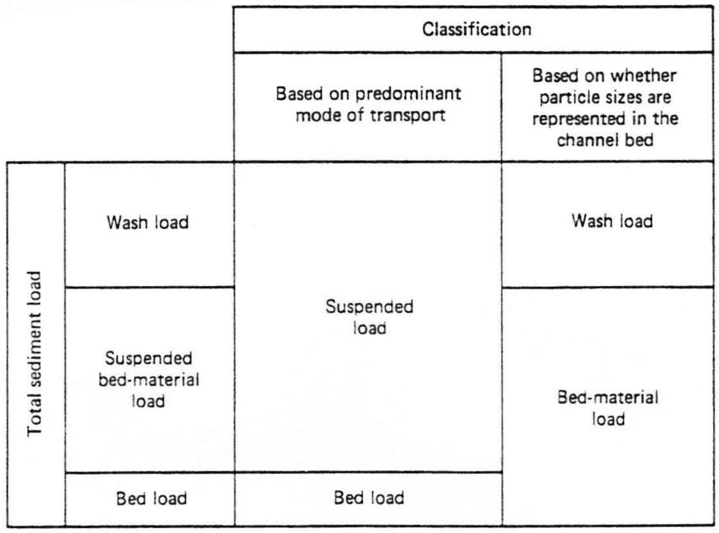

Figure 15-5 Relationship between the two classifications of sediment load [11].

There is a threshold value of bottom shear stress above which the particles actually begin to move. This threshold value is referred to as *critical bottom shear stress*, or critical tractive stress. Determinations of critical tractive stress for given flow and sediment conditions are largely empirical in nature. The Shields curve, shown in Fig. 15-6, represents the earliest attempt to combine theoretical and empirical approaches to estimate critical tractive stress [2]. The Shields curve depicts the threshold of motion, i.e., the condition separating motion (above the curve) from no motion (below the curve) [36].

The abscissa in the Shields diagram is the boundary Reynolds number, defined as:

$$R_* = \frac{U_* d_s}{\nu} \tag{15-15}$$

in which R_* = boundary Reynolds number; U_* = shear velocity; d_s = mean particle diameter; and ν = kinematic viscosity of water. The shear velocity is defined as

$$U_* = \left(\frac{\tau_o}{\rho}\right)^{1/2} \tag{15-16}$$

in which ρ = density of water. The ordinate in the Shields diagram is the dimensionless tractive stress, defined as:

$$\tau_* = \frac{\tau_o}{(\gamma_s - \gamma)d_s} \tag{15-17}$$

in which τ_* = dimensionless tractive stress.

The Shields diagram indicates that, within a midrange of boundary Reynolds numbers (approximately 2–200), the dimensionless critical tractive stress can be taken as a constant for practical purposes. Therefore, within this range, the critical tractive stress is proportional to the sediment particle size. For instance, assuming a value of

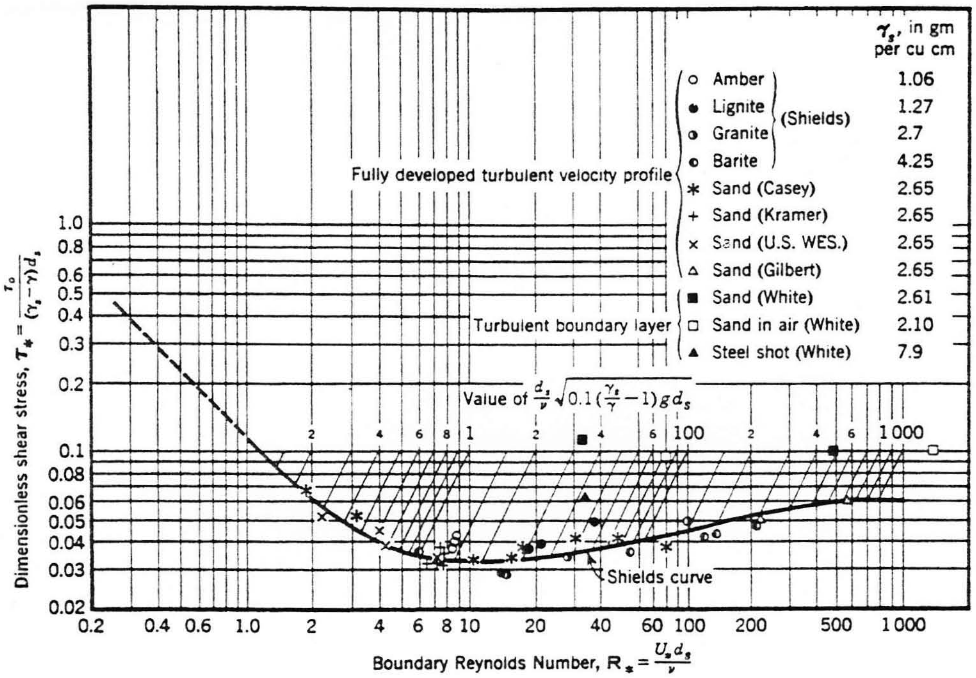

Figure 15-6 Shields diagram for initiation of motion [2].

dimensionless critical tractive stress $\tau_* = 0.04$ (from the Shields diagram, Fig. 15-6), Eq. 15-17 leads to:

$$\tau_c = 0.04(\gamma_s - \gamma)d_s \tag{15-18}$$

in which τ_c = critical tractive stress. For quartz particles ($\gamma_s = 2.65 \times 62.4$ lb/ft^3), Eq. 15-18 reduces to:

$$\tau_c = 0.34 d_s \tag{15-19}$$

in which critical tractive stress is given in pounds per square foot and particle diameter in inches. Extensive experimental studies by Lane [29] have shown that the coefficient in Eq. 15-19 is around 0.5. Lane, however, used the d_{75} particle size (i.e., the diameter for which 75% by weight is finer) instead of the mean diameter (d_{50}) used by Shields.

Example 15-5.

Based on the Shields criterion for initiation of motion, determine whether a 3-mm diameter quartz particle is at rest or moving under the action of a 7-ft flow depth with channel slope $S_0 = 0.0001$. Assume water temperature 70°F.

From Eq. 15-14, the bottom shear stress is: $\tau_o = 62.4 \times 7.0 \times 0.0001 = 0.0437$ lb/ft^2. From Eq. 15-17, the dimensionless tractive stress is $\tau_* = 0.0437/[(2.65 - 1.0) \times 62.4 \times 3.0/(25.4 \times 12)] = 0.0431$. From Eq. 15-16, the shear velocity is $U_* = (0.0437/1.94)^{1/2} = 0.150$ ft/s. From Table A-2, the kinematic viscosity is 1.058×10^{-5} ft^2/s. From Eq. 15-15, the boundary Reynolds number is $R_* = 0.150 \times [3.0/(25.4 \times 12)]/(1.058 \times 10^{-5}) = 140$. For $R_* = 140$, the dimensionless critical tractive stress is obtained from the

Shields curve (Fig. 15-6): $\tau_{*_c} = 0.048$. Since $\tau_* = 0.0431$ is less than $\tau_{*_c} = 0.048$, it is concluded that the particle is at rest.

Forms of Bed Roughness. Streams and rivers create their own geometry. In particular, alluvial rivers determine to a large extent their cross-sectional shape and boundary friction as a function of the prevailing water and sediment discharge. An inherent property of river flows is their tendency to minimize changes in stage caused by changes in discharge. This is accomplished through a continuous adjustment in boundary friction in such a way that high values of friction prevail during low flows, while low values of friction prevail during high flows [26].

Bed forms are three-dimensional configurations of bed material, which are formed in streambeds by the action of flowing water. Adjustments in boundary friction are made possible by the existence of these bed forms, which develop during low flows (lower flow regime) only to be obliterated during high flows (upper flow regime) [37]. Boundary friction consists of two parts: (1) grain roughness and (2) form roughness. Grain roughness is a function of particle size; form roughness is a function of size and extent of bed forms. Grain roughness is essentially a constant when compared to the variation in form roughness that can be attributed to bed forms.

Studies have shown that several forms of bed roughness can exist on river bottoms, depending on the energy and bed-material transport capacity of the flow. In the absence of sediment movement, the bed configuration is that of plane bed with no sediment motion. With sediment movement, the following forms of bed roughness have been identified: (1) ripples, (2) dunes and superposed ripples, (3) dunes, (4) washed-out dunes or transition, (5) plane bed with sediment motion, (6) antidunes, and (7) chutes and pools. Sketches of these bed configurations are shown in Fig. 15-7 [37].

The occurrence of different forms of bed roughness can be shown to be related to the median fall diameter of the particles forming the bed and to the *stream power* of the flow. Stream power is defined as the product of bottom shear stress and mean velocity. Such a relationship is shown in Fig. 15-8. For low values of stream power, i.e., below the critical tractive stress, there is no bed load transport, and the streambed remains essentially flat. This is the condition of plane bed with no sediment motion. An increase in stream power leads first to ripples, then to dunes with superposed ripples, and, subsequently, to dunes. Ripples, however, are a rare occurrence for sediments coarser than 0.6 mm. Dunes are longer and bigger than ripples and occur at flow velocities and sediment loads that are generally greater than those of ripples. The plane bed with sediment motion represents the condition at which the flow's stream power is large enough to obliterate the dunes, essentially eliminating the form roughness. The plane bed, then, represents the condition of minimum boundary friction. For high values of stream power, antidunes (upper regime) form in conjunction with surface waves, with the tendency for upstream movement, usually under supercritical flow conditions. For even higher values of stream power, e.g., in very steep streams, the bed configuration resembles a sequence of chutes and pools [37].

The assessment of bed-form type has practical implications for engineering hydrology. Bed forms determine boundary roughness; in turn, boundary roughness determines river stages. For instance, ripples are associated with values of Manning n in the range 0.018 to 0.030, with form roughness usually a fraction of grain roughness.

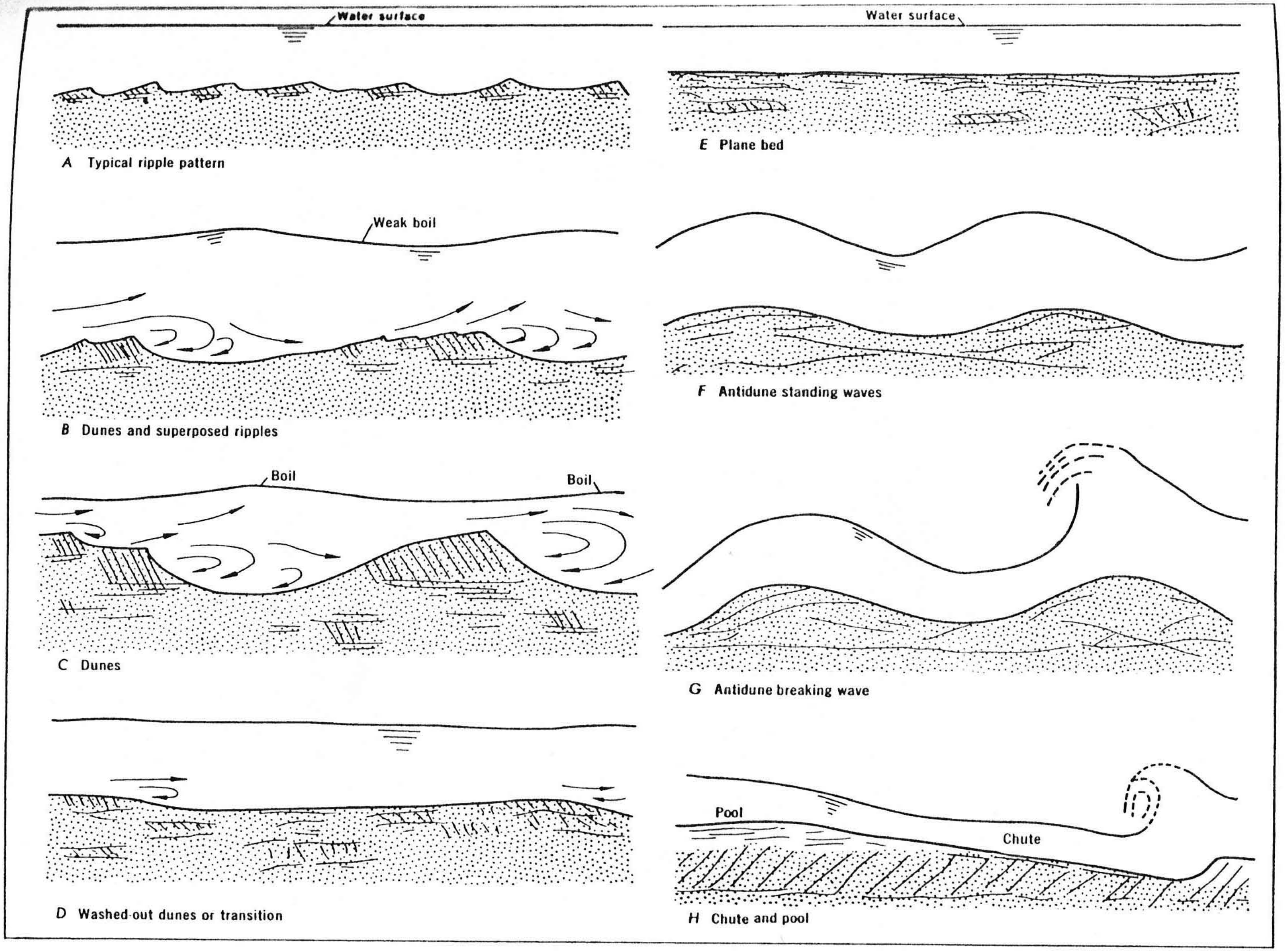

Figure 15-7 Forms of bed roughness in alluvial channels [37].

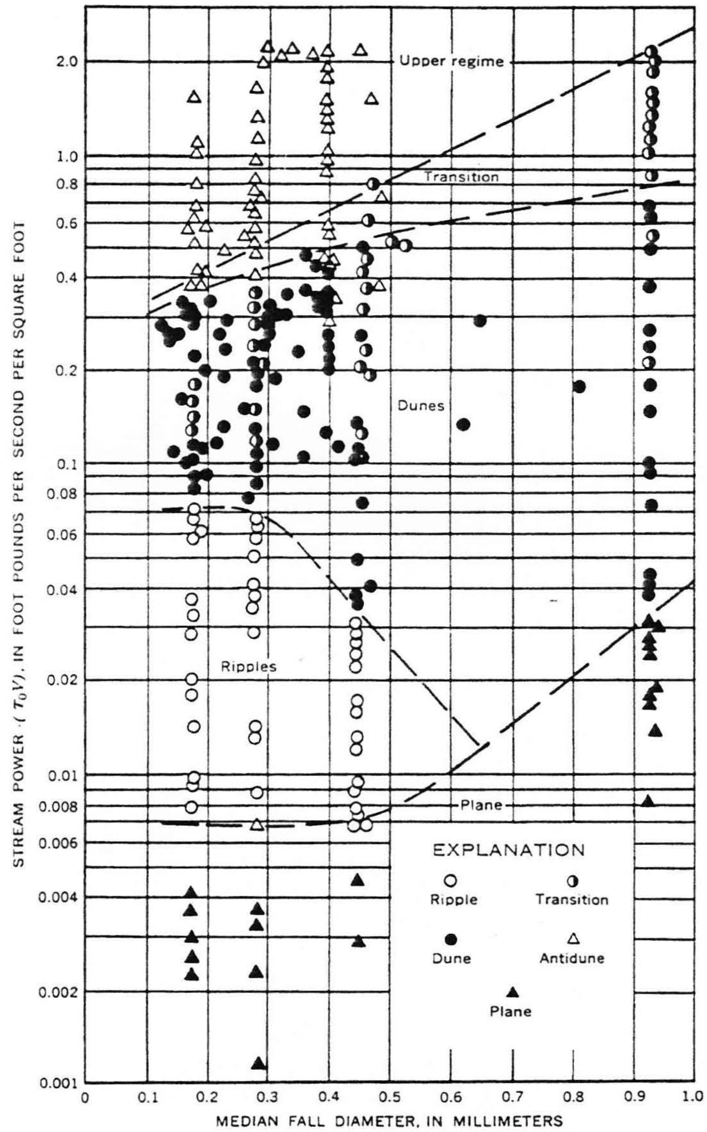

Figure 15-8 Form of bed roughness versus stream power and median fall diameter of bed material [37].

Dunes, however, are associated with n values in the range 0.020 to 0.040, with form roughness of the same order as grain roughness. Moreover, plane bed with sediment motion is associated with relatively low n values, in the range 0.012 to 0.015, and in this case, form roughness is minimal.

The proper assessment of boundary friction, including its variation as the flow changes from lower regime (ripples, ripples on dunes, and dunes) to upper regime (plane bed with sediment movement, antidunes, and chutes and pools) is an important subject in engineering hydrology and fluvial hydraulics.

Concentration of Suspended Sediment. For a given volume of water-sediment mixture, the suspended-sediment concentration is the ratio of the weight of dry sediment to the weight of the water-sediment mixture, expressed in parts per million. To convert the concentration in ppm to milligrams per liter (mg/L), the applicable factor ranges from 1.0 for concentrations between 0 and 15,900 ppm, to 1.5 for concentrations between 529,000 and 542,000 ppm, as shown in Table 15-8.

The suspended-sediment concentration varies with the flow depth, usually being higher near the stream bed and lower near the water surface. The coarsest sediment fractions, typically those in the sand size, exhibit the greatest variation in concentration with flow depth. The finer fractions, i.e., silt and clay particles, show a tendency for a nearly uniform distribution of suspended-sediment concentration with flow depth.

TABLE 15-8 FACTOR TO CONVERT CONCENTRATION IN PARTS PER MILLION (ppm) TO MILLIGRAMS PER LITER (mg/L) [19]

(1) $\text{ppm} = \dfrac{\text{Weight of Dry Sediment}}{\text{Weight of Water-and-Sediment Mixture}} \times 10^6$	(2) Factor
0-15,900	1.00
16,000-46,900	1.02
47,000-76,900	1.04
77,000-105,000	1.06
106,000-132,000	1.08
133,000-159,000	1.10
160,000-184,000	1.12
185,000-209,000	1.14
210,000-233,000	1.16
234,000-256,000	1.18
257,000-279,000	1.20
280,000-300,000	1.22
301,000-321,000	1.24
322,000-341,000	1.26
342,000-361,000	1.28
362,000-380,000	1.30
381,000-398,000	1.32
399,000-416,000	1.34
417,000-434,000	1.36
435,000-451,000	1.38
452,000-467,000	1.40
468,000-483,000	1.42
484,000-498,000	1.44
499,000-513,000	1.46
514,000-528,000	1.48
529,000-542,000	1.50

Note: To obtain concentration in mg/L, multiply concentration in ppm by applicable factor in Col. 2. This table is based on density of water 1 g/mL, plus or minus 0.005, in the temperature range 0°C-29°C, specific gravity of sediment 2.65, and dissolved solids concentration in the range 0 to 10,000 ppm.

Figure 15-9 shows the variation of suspended-sediment concentration along the flow depth. In this figure, y is the fraction of flow depth measured from the channel bottom, a is the reference distance measured from the channel bottom, and d is the flow depth. The abscissas show the dimensionless ratio C/C_a, in which C_a is the sediment concentration at the reference distance and C is the sediment concentration at a distance $y - a$. The value of a is small compared to d. The plot of Fig. 15-9 is specifically for the case of $a/d = 0.05$. The ordinates show the dimensionless ratio $(y - a)/(d - a)$. The curve parameter z is the Rouse number, defined as

$$z = \frac{w}{\beta \kappa U_*} \tag{15-20}$$

in which z = Rouse number (dimensionless); w = fall velocity of sediment particles; β = a coefficient relating mass and momentum transfer ($\beta \cong 1$ for fine sediments); κ = von Karman's constant ($\kappa = 0.4$ for clear fluids); and U_* = shear velocity, Eq. 15-16. From Fig. 15-9, it is seen that for high Rouse numbers the variation of suspended-sediment concentration along the flow depth is quite marked. Conversely, for low Rouse numbers there is a tendency for greater uniformity of suspended-sediment concentration along the flow depth.

Sediment Transport Prediction

Sediment load, sediment discharge, and sediment transport rate are synonymous in practice. However, bed load, suspended bed-material load, and wash load are mutually exclusive. Sediment transport prediction refers to the estimation of sediment transport rates under equilibrium (i.e., steady uniform) flow conditions.

There are numerous formulas for the prediction of sediment transport [2]. Most formulas compute only bed-material load, consisting of bed load and suspended bed-material load. A few compute total sediment load, which consists of bed load, suspended bed-material load, and wash load. Yet some may compute bed load and suspended bed-material load separately. Invariably, sediment transport formulas have some empirical components and, therefore, are most applicable within the range of laboratory and/or field data used in their development.

Duboys Formula. The Duboys formula is widely recognized as one of the earliest attempts to develop a sediment transport predictor. The Duboys formula is [14]:

$$q_s = \Psi_D \tau_o (\tau_o - \tau_c) \tag{15-21}$$

in which q_s = bed-material transport rate per unit channel width, in pounds per second per foot; Ψ_D = a parameter that is a function of particle size in cubic feet per pound per second; τ_o = bottom shear stress in pounds per square foot; and τ_c = critical tractive stress in pounds per square foot. Values of Ψ_D and critical tractive stress for use in the Duboys equation are shown in Fig. 15-10 [3].

Example 15-6.

Given a channel of mean flow depth $d = 12$ ft, mean width b = 320 ft; equilibrium channel slope $S_0 = 0.0001$, and median particle size $d_{50} = 0.6$ mm, calculate the bed-material transport rate by the Duboys formula.

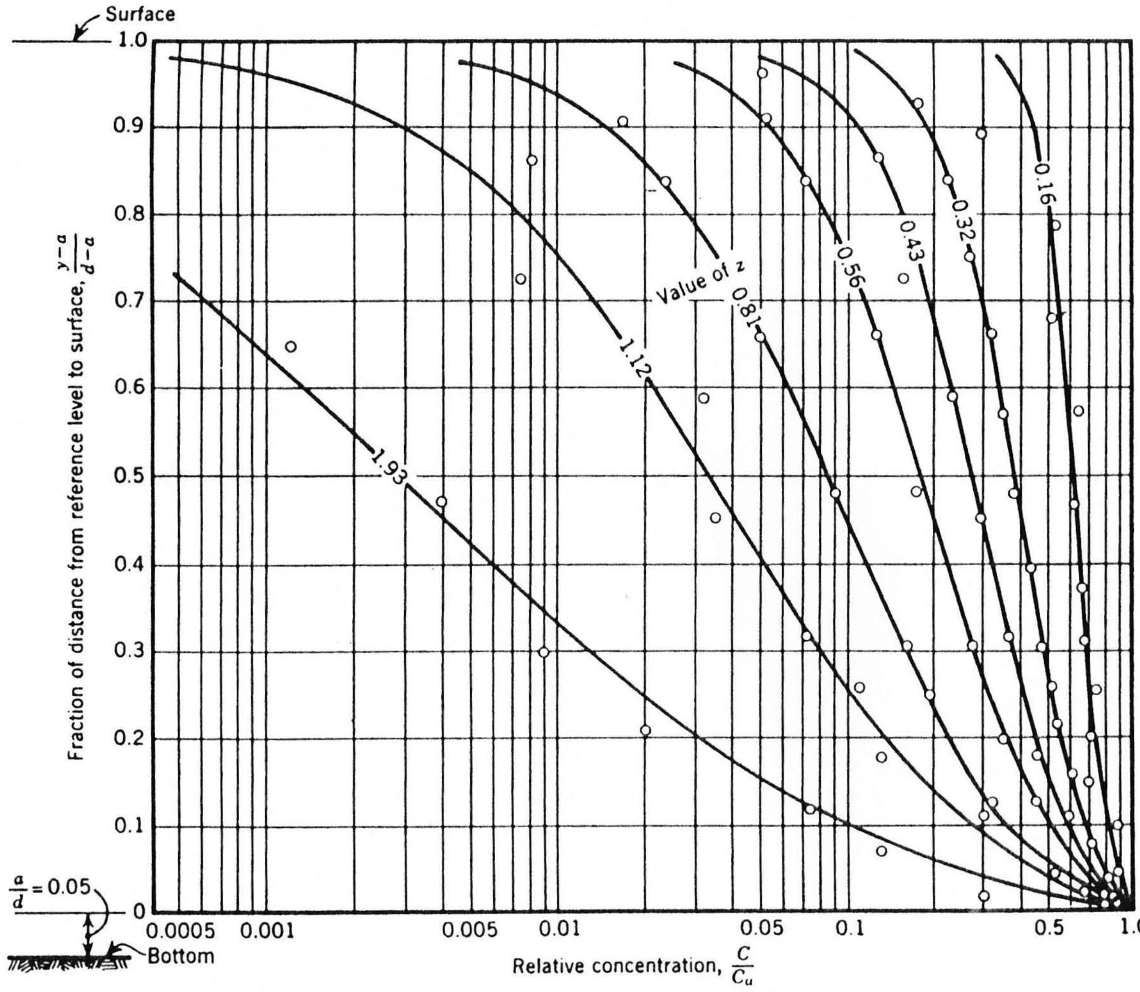

Figure 15-9 Variation of suspended-sediment concentration along the flow depth [2].

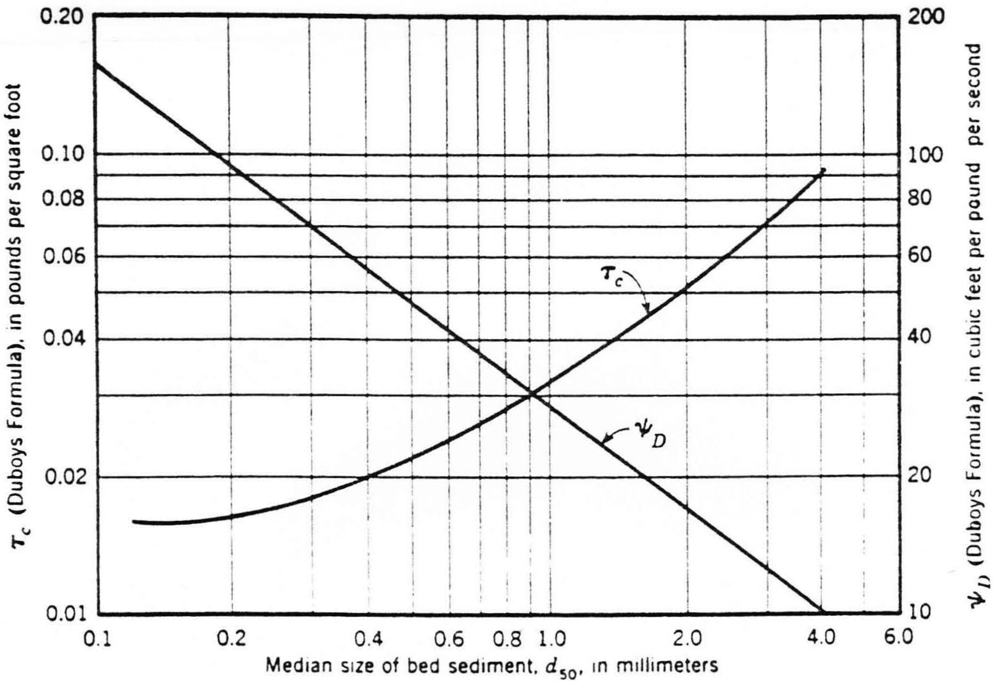

Figure 15-10 Values of Ψ_D and τ_c for use in the Duboys equation [2].

From Eq. 15-14, $\tau_o = 62.4 \times 12.0 \times 0.0001 = 0.07488$ lb/ft^2. From Fig. 15-10, $\Psi_D = 42$ ft^3/lb/s; $\tau_c = 0.025$ lb/ft^2. From Eq. 15-21, $q_s = 42 \times 0.07488 \times (0.07488 - 0.025) = 0.157$ lb/s/ft. Therefore, $Q_s = q_s b = 0.157 \times 320 = 50.2$ lb/s.

Meyer-Peter Formula. The development of the Meyer-Peter formula was based on flume data, with uniform bed material size in the range 3 to 28 mm. Such coarse sediments do not produce appreciable bed forms; therefore, the formula is applicable to coarse sediment transport where form roughness is negligible. The Meyer-Peter formula is [2, 33]:

$$q_s = (39.25 q^{2/3} S_o - 9.95 d_{50})^{3/2} \tag{15-22}$$

in which q_s = bed-material transport rate per unit channel width in pounds per second per foot; q = water discharge per unit channel width in cubic feet per second per foot; S_0 = equilibrium channel slope; and d_{50} = median particle size in feet.

Example 15-7.

Given a channel of mean flow depth $d = 2$ ft; mean width $b = 25$ ft; mean velocity $v = 6$ fps; channel slope $S_0 = 0.008$; median particle size $d_{50} = 22$ mm, calculate the bed-material transport rate by the Meyer-Peter formula.

Discharge per unit width is $q = vd = 6 \times 2 = 12$ ft^3/s/ft. From Eq. 15-22, $q_s = \{(39.25 \times 12^{2/3} \times 0.008) - [9.95 \times 22/(25.4 \times 12)]\}^{3/2} = 0.894$ lb/s/ft. Therefore, $Q_s = q_s b = 22.35$ lb/s.

Einstein Bed-load Function. In 1950, Einstein published a procedure for the computation of bed material transport rate by size fractions [15]. The method was developed based on theoretical considerations of turbulent flow, supported by laboratory and field data. Einstein is credited with the introduction of several novel concepts in sediment transport theory, including the separation of boundary friction into grain and form roughness and the use of statistical properties of turbulence to explain the mechanics of sediment transport.

Einstein's bed-load function first computes the bed-load transport rate. Then it uses the bed-load transport rate to aid in the integration of the product of the suspended sediment concentration profile and the flow velocity profile, to determine the suspended bed-material transport rate, per individual size fraction. Several step-by-step procedures have been reported in the literature [2].

Modified Einstein Procedure. The modified Einstein procedure was developed by Colby and Hembree [6] in order to include actual measurements of suspended load into the framework of the original Einstein method. Typically, measurements of suspended load do not include (a) the bed load and (b) the fraction of suspended load moving too close to the streambed to be effectively sampled. The modified Einstein procedure calculates the total sediment load by size fractions based on measurements of suspended load and relevant geometric and hydraulic characteristics of the stream or river. Details of the method are reported in the literature [2, 6].

Colby's 1957 Method. Colby's 1957 method is based on some of the same measurements that led to the development of the modified Einstein procedure. However, unlike the latter, it does not account for sediment transport rate by size fractions. Instead, it provides the total bed-material discharge, i.e. the sum of measured and unmeasured bed-material discharges.

The following data are needed in an application of the Colby 1957 method: (1) mean flow depth d, (2) mean channel width b, (3) mean velocity v, and (4) measured concentration of suspended bed-material discharge C_m. The procedure is as follows [7]:

1. Use Fig. 15-11 to obtain the uncorrected unmeasured sediment discharge q_u' (in tons per day per foot of width) as a function of mean velocity.
2. Use Fig. 15-12 to obtain the relative concentration of suspended sands C_r (in parts per million) as a function of mean velocity and flow depth.
3. Calculate the availability ratio by dividing the measured concentration of suspended bed-material discharge C_m (ppm) by the relative concentration of suspended sands C_r (ppm).
4. Use the mean line of Fig. 15-13 and the availability ratio to obtain the correction factor C to be multiplied by the uncorrected unmeasured sediment discharge q_u' to obtain the unmeasured sediment discharge q_u (in tons per day per foot).
5. The total bed-material discharge q_s is the sum of measured and unmeasured sediment discharges:

$$q_s = 0.0027\ C_m q + q_u \tag{15-23}$$